On the Performance of TPC-based STBC Coded MIMO-OFDM System over IMT2000 Channels

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***Abstract*—Space-Time Block Code (STBC) provides full diversity gain but does not achieve any coding gain. To provides a significant coding gain ,STBC needs to be concatenated with an outer code. Turbo Product Code (TPC) is a kind of high-efficient coding scheme with low latency decoding.Moreover,TPC does not have an error floor. In this paper, we investigate the performance of a TPC-based STBC coded MIMO-OFDM system under several IMT2000 channel environments.**

I. INTRODUCTION

In the last decade, much research effort has been spent in vestigating multiple-input multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM) systems. In a frenquency selective fading channel environment, MIMO systems are damaged due to the intersymbol interference. Thus, MIMO has been proposed integrating with OFDM [1-2] and applying to broadband communications. An IMT2000 channel belongs to frequency selective fading channel. Among each path of an IMT2000 channel, the delay and the average power are different.

Space-time block code (STBC)[3-4] can obtain the full diversity gain. Turbo product code (TPC)[5] can get coding gain. Hence, we concatenate STBC with TPC to improve diversity gain as well as coding gain. Considering the cost and the bulk, we think MIMO systems with two-transmit antennas (2Tx) and two-receive antennas (2Rx) are more worth studying in the near future.

Although TPC isn't a novel forward error correction (FEC) approach, it is an advanced coding one. TPC integrates some merits of turbo code and product code. In terms of encoding, the TPC belongs to product code and from the decoding perspective, the TPC is an extension of turbo code. TPC is a multidimensional array of block codes and is a large code built from two, three, or many smaller block codes. At present, TPC has been applied to the satellite communication system by Comtech AHA corporation[6].

All existing literatures mainly focus on the combination of turbo code and STBC[7-10] as well as that of turbo code and BLAST[11].In this paper, we investigate the performance of a TPC-based STBC coded MIMO-OFDM system under several IMT2000 channel environments. Since TPC contains an inherent row-column interleaving process, block interleaver is not required. We select all kinds of extended Hamming code as TPC's constituent code, including (8,4), (16,11),(32,26) , (64,57) as well as (128,120).

This paper is organized as follows. In Section Ⅱ,we analyse the channel capacity properties of TPC-based STBC coded MIMO-OFDM system. Performance bounds of TPC are induced in Section Ⅲ. In Section Ⅳ, Computer simulation results are presented. Conclusions are drawn in section Ⅴ.

II. THE TPC-BASED STBC CODED MIMO-OFDM SYSTEM MODEL AND CHANNEL CAPACITY PROPERTIES

The TPC-based STBC coded MIMO-OFDM system model with 2Tx and 2Rx is shown in Fig. 1. Without loss of generality, we consider the wireless system with M transmit antennas and L receive antennas and evaluate its channel capacity properties. Perfect channel state information is assumed available at receiver, but the channel characteristics are unknown at the transmitter. We make use of the IMT2000 channel model, which is assumed to be quasi-static so that the channel characteristics remain similar within two continuous time slots.Since a subchannel in STBC coded MIMO-OFDM system with M transmit antennas and L receive antennas is equivalent to $(1,ML)$-receive diversity system employing maximum ratio combination (MRC) ,we firstly analyse $(1,K)$-receive diversity system employing maximum ratio combination.

In a bandlimit additive white Gaussian noise (AWGN) channel, channel capacity after the normalized bandwidth can be expressed as

$$C = \log_2^{(1+SNR)}\ bits/\mathrm{sec}/Hz \qquad (1)$$

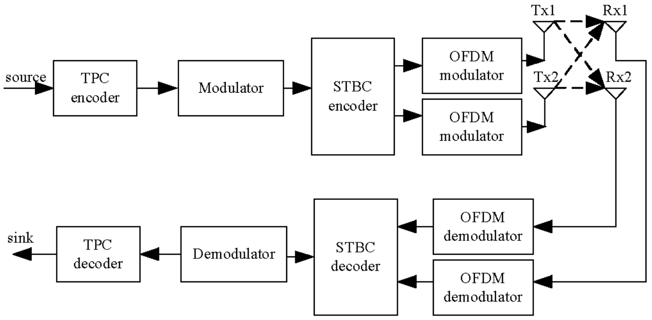

Figure 1. The TPC-based STBC coded MIMO-OFDM system model with 2-transmit and 2-receive antennas

where SNR is the signal to noise ratio.

Since the SNR varies with time for a Rayleigh fading channel, only the average channel capacity can be computed. For a flat Rayleigh fading channel, the received signals with receive diversity K can be written as

$$r_j(t) = h_j(t)s(t) + n_j(t) \quad j = 1, 2...., K \tag{2}$$

where $h_j(t)$ denotes complex gaussian random variables with zero mean and variance 0.5 per dimension. This is equivalent to the assumption that each path between a transmit and a receive antenna has frequency-flat independent Rayleigh fading. $s(t)$ is the transmitted baseband signal with power P. $n_j(t)$ represents AWGN with variance N_0 at the receive antenna j . $h_j(t)$ can be given by

$$h_j(t) = \alpha_j(t)[\cos\omega_j(t) + i\sin\omega_j(t)] \quad ,i=\sqrt{-1} \tag{3}$$

The output of MRC for the receive signals is

$$\tilde{s} = \sum_{j=1}^{K} r_j(t)[h_j(t)]^* = s(t)\sum_{j=1}^{K}\left|\alpha_j(t)\right|^2 + \sum_{j=1}^{K}[h_j(t)]^* n_j(t) \tag{4}$$

where $(.)^*$ denotes the complex conjugate.

The instantaneous SNR at receiver at time t is

$$\Psi(t) = P \times \sum_{j=1}^{K}\left|\alpha_j(t)\right|^2 / N_0 \tag{5}$$

The average SNR is defined by

$$\Gamma = E(\Psi(t)) = P \times K / N_0 \tag{6}$$

Moreover,the Probability Density Function(PDF) of the instantaneous SNR is given by

$$p(\Psi) = \frac{1}{(K-1)!} \times \left(\frac{K}{\Gamma}\right)^K \times \Psi^{K-1} \times \exp(-\frac{K}{\Gamma} \times \Psi) \tag{7}$$

Thus, normalized average channel capacity is obtained by substituting (7) into the following equation

TABLE I. AVERAGE CHANNEL CAPACITY AND CAPACITY OF GAUSSIAN CHANNEL

Γ (dB)	**Normalized Average Channel Capacity**				**Capacity of Gaussian Channel**
	K=1	K=2	K=3	K=4	
0	0.86	0.9229	0.9572	0.9773	1
5	1.71	1.87	1.9571	2.0058	2.057
10	2.9032	3.16	3.3094	3.3848	3.5
15	4.32	4.6768	4.8536	4.94	5.03
20	5.88	6.2836	6.4771	6.5705	6.66
25	7.4946	7.92	8.1228	8.2184	8.3
30	9.1433	9.5779	9.7837	9.8756	9.96
35	10.7894	11.23	11.443	11.536	11.628

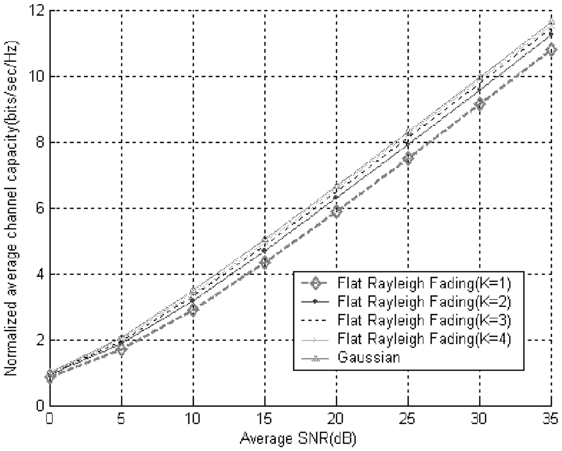

Figure 2. Normalized average channel capacity at receive diversity K

$$C_{av} = \int_0^{\infty} \log_2^{(1+\Psi)} p(\Psi) d\Psi \tag{8}$$

In Fig.2, we plot the capacity of Gaussian channel as well as normalized average channel capacity for the Rayleigh channel with different diversity gains, respectively. It can be observed that as the diversity gain K increases, the channel capacity for the Rayleigh channel approaches that for the Gaussian channel. Based on the observation, we conclude that for a STBC coded MIMO-OFDM system with M transmit antennas and L receive antennas, when the product of M and L is large enough, the frequency response of each OFDM subchannel can be model as solely by an Gaussian channel .Comparing a transmission rate of practical system to normalized average channel capacity as shown in Fig. 1,we can obtain some sense knowledge to help us evaluate a wireless communication system.

III. PERFORMANCE BOUNDS OF TPC

Assuming that a linear block code $c = (n, k, \delta)$ is applied to a digital communication system employing BPSK over an AWGN channel with noise power spectral density $N_0/2$,the probability of bit error P_b obtained by maximum likelihood decoding[12] is upperbounded by [13]

$$P_b \le \sum_{\omega=\delta}^{n} \frac{\omega}{n} A_\omega(c) Q\left(\sqrt{\frac{2E_b}{N_0} R\omega}\right) \tag{9}$$

where E_b and R are the information bit energy and code rate, respectively; $A_\omega(c)$ represents the weight enumerator of c and is the total number of codewords with Hamming weight equal ω;and $Q(.)$ is the Q-function[12].

For a two-dimensional TPC consisting of code $c^1 = (n_1, k_1, \delta_1)$ and $c^2 = (n_2, k_2, \delta_2)$, the maximum likelihood asymptote(MLA) can be written as[14]

$$P_{MLA} = \frac{\delta_1 \delta_2}{n_1 n_2} A_{\delta_1}(c^1) A_{\delta_2}(c^2) Q\left(\sqrt{\frac{2E_b}{N_0} R_1 R_2 \delta_1 \delta_2}\right) \tag{10}$$

where $R_1 = k_1 / n_1$ and $R_2 = k_2 / n_2$ are code rates of c^1 and c^2 , respectively. δ_1 and δ_2 is the minimum distance of the corresponding codes. This asymptote is obtained by considering codewords that have a minimum Hamming weight.It has been shown that the weight enumerating equation for a Hamming code is given by[15]

$$A_m = \frac{1}{n+1}\left\{ C_n^m + n(-1)^{\left\lfloor \frac{m+1}{2} \right\rfloor} C_{\left\lfloor \frac{n-1}{2} \right\rfloor}^{\left\lfloor \frac{m}{2} \right\rfloor} \right\} \tag{11}$$

where n is the code length, m is the code weight, $C_n^m = \frac{n!}{m!(n-m)!}$,and $\lfloor X \rfloor$ takes the nearest integer that is less than or equal to X.

Using (11),we can easily get the weight distribution of a Hamming code. For example, for the (15,11) Hamming code,

$$A_3 = \frac{1}{16}[C_{15}^3 + 15(-1)^2 C_7^1] = 35 \tag{12}$$

$$A_4 = \frac{1}{16}[C_{15}^4 + 15(-1)^2 C_7^2] = 105 \tag{13}$$

The weight distribution of an extended Hamming code is

$$\left\{A'_m\right\} = \left\{1, 0, 0, 0, A_3 + A_4, 0, A_5 + A_6, 0, \ldots, A_{n-1} + A_n\right\} \tag{14}$$

For the (16,11) extended Hamming code, because its Hamming weight m equals 4 ,we obtain

$$A'_4(16,11) = A_3(15,11) + A_4(15,11) = 35 + 105 = 140 \tag{15}$$

Similarly, for the extended Hamming codes (8,4), (32,26), (64,57) and (128,120), we have

$$A'_4(8,4) = A_3(7,4) + A_4(7,4) = 7 + 7 = 14 \tag{16}$$

$$A'_4(32,26) = A_3(31,26) + A_4(31,26) = 155 + 1085 = 1240 \tag{17}$$

$$A'_4(64,57) = A_3(63,57) + A_4(63,57) = 651 + 9765 = 10416 \tag{18}$$

$$\begin{aligned} A'_4(128,120) &= A_3(127,120) + A_4(127,120) \\ &= 2667 + 82677 = 85344 \end{aligned} \tag{19}$$

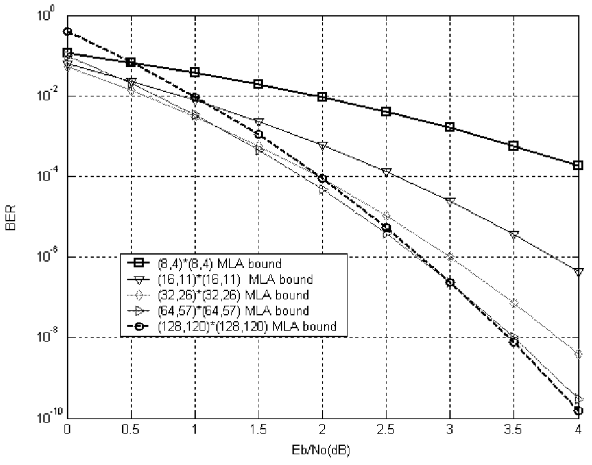

Figure 3. Maximum likelihood asymptote bound of typical TPC

If the row and column component codes of the TPC are identical ,we compute the MLA (BER) for the two-dimensional TPC by substituting $\delta_1 = \delta_2 = 4$ into (9).Some of the results are plotted in Fig. 3.

IV. SIMULATION RESULTS

The parameters of the TPC-based STBC coded MIMO-OFDM system are set as follows.

1) Fast Fourier Transform (FFT) block size is 1024.
2) Cyclic prefix interval is 64.
3) Guard interval is 128.
4) Carrier frequency is 2GHz.
5) Sampling Frequency is 100MHz.
6) The signal power per transmit antenna equals unity.
7) BPSK modulation scheme is used.
8) Speed of mobile over IMT2000 pedestrian channels equals 5km/hr.
9) Speed of mobile over IMT2000 vehicular channels equals 100km/hr.
10) Noise power is defined as $2E_p \times 10^{-0.1SNR}$,where E_p denotes signal power per transmit antenna, SNR is the signal to noise ratio at receiver, i.e E_s / N_0 .
11) Default Doppler Frequency is 185.3705557Hz.

We then simulate the system with different TPC code rates over the IMT2000 channels. The bit error rate results are shown in Figs.4-6. These points on the right side of the solid vertical lines correspond to zero BER in Fig. 4.From the curves, we can observe that among the five TPCs under investigation, the (8,4)*(8,4) TPC provides the best performance in the IMT2000 channels. The disadvantage of the (8,4)*(8,4) TPC is that the code rate is very low. The code rate of the (128,120)*(128,120) TPC is the highest but the performance is the worst. To compromise code rate with performance, it is therefore more important for practical applications to have (16,11)*(16,11), (32,26)*(32,26) or

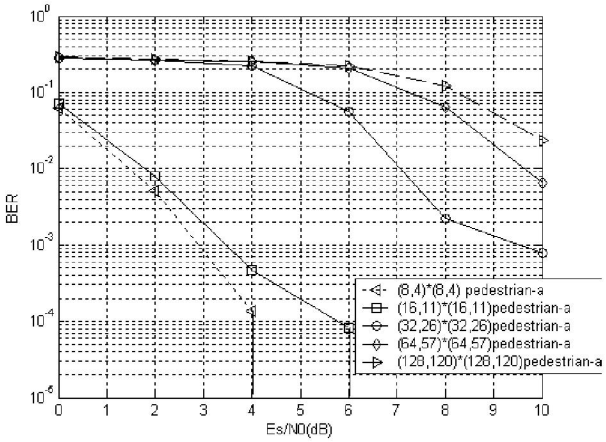

Figure 4. Performance of TPC-based STBC coded MIMO-OFDM system over IMT2000 pedestrian-*a*

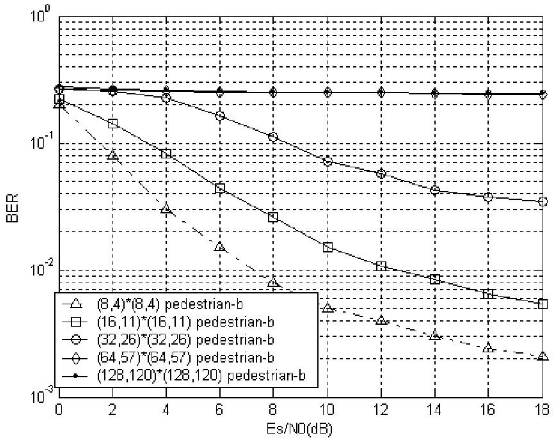

Figure 5. Performance of TPC-based STBC coded MIMO-OFDM system over IMT2000 pedestrian-*b*

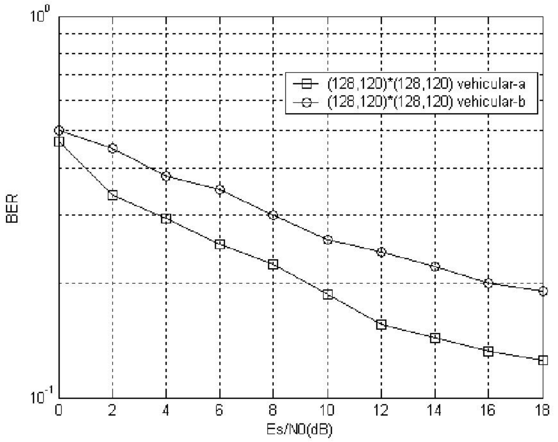

Figure 6. Performance of TPC-based STBC coded MIMO-OFDM system over IMT2000 vehicular-*a* and *b* channels

(64,57)*(64,57) TPCs used in beyond 3G mobile communication systems.

V. CONCLUSIONS

In this paper, we introduce TPC-based STBC coded MIMO-OFDM system model and channel capacity properties of STBC coded MIMO-OFDM system .We then induce performance bounds of TPC. These related background knowledge is essential to help us design our proposed system. Finally, we study the performance of a TPC-based STBC coded MIMO-OFDM system over IMT2000 channels. We find that for an STBC coded MIMO-OFDM system with M transmit antennas and L receive antennas, when the product of M and L is large enough, the frequency response of each OFDM subchannel can be modeled solely by an AWGN channel. We also conclude that to provide a satisfactory bit error rate performance of the TPC-based STBC coded MIMO-OFDM system,the code rate cannot be too large. Possible TPC codes that can be used include (16,11)*(16,11),(32,26)*(32,26) and (64,57)*(64,57).

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